Chapter 4

Inventory Management

4.1 Introduction

■ An Inventory consists of usable but idle (not used immediately) resources such as men, machines, materials or money. Inventory is very important for a company as it directly connected to the revenew. If the resource involved is a material, the inventory is also called 'stock'. There are mainly three types of inventory: Raw materials, Work in progress and finished goods.

► Raw Materials: Items use to make your finished goods.

► Work in Progress: Items in the process of making finished goods for sales.

► Finished Goods: The products you sell to your customers.

■ Suppose for the wooden furniture company Godrej Interio: The wood bought from the supplier to make furniture are raw materials. All unfinished parts before the assembly of the finished product are work in progress inventory. It including labor, shape cutting, assemble with gun, design, polish, etc. The finished goods are ready for sale product as we see in the showroom.

► We will discuss a detailed classification of inventories later.

4.1.1 Requirement of Inventory

■ Although inventory is an idle resouces but most of the companies retain that for effective and smooth performance. Lack of inventory causes various trouble for an organisation and ultimately hinder the profit.

► Suppose a company has no inventory, then after receiving an order they have to purchase and recieve raw materials, then start production. It's a time consuming process makes the customers wait for a long delivery time of the products, may causes to lose the customer.

▶ Depending on the type, most of companies allocate a certain proportion of its budget for the inventory and its management.

■ Inventory management is the process of tracking and controlling the business inventory as it is bought, manufactured, stored and used. It manages the complete progress of inventories from purchasing to sale. It also determines that based on the need, the right quantities of the right item in the right location at the right time are always available.
▶ At a basic level, inventory management works by tracking products, components and ingredients across suppliers, stock on hand, production and sales to ensure that stock is used as efficiently and effectively as possible. This management process will get complex as we consider various dimensions in it. Also most of the companies try to improve their existing inventory management process.
▶ The application of operations research techniques in this area is providing a powerful tool for managing inventories and it's known as scientific inventory management.

Question 4.1. *Why it is essential for mantaining an inventory for almost all the companies?*

★ *There are several reasons behind the need of inventory management:*

1. *It helps the company to flow in a smooth and efficient way.*
2. *Reduces the waiting time for a customer which leads to customer satisfaction and future growth.*
3. *Maintaining inventory requires bulk purchasing which is cost effective. Also it takes the advantage of market price i.e. if there is no inventory then company has to purchase the inventory after the order received and it may possible that product price is high on that time.*
4. *Reduces product cost due to uninterrupted flow. Inventories also plays a critical role due to product scarcity (or, slow raw material received) at the market.*
5. *For long distance customers, inventories saved a significant amount of time. For example customer transite time is six days and if there is an inventory then product making requires small amount of time and the customer received the order quickly.*
6. *Suppose a customer transite time is four days and they just created the order before journey. So if the company has inventory, then it may possbile to create the required product within four days.*

■ Note that, efficient inventory management is required else it will impact heavily on the company profit.

4.1.2 Classification of Inventories

Inventories are mainly classified into two categories: Direct and Indirect.

Direct Inventories: Inventories directly used for production and **are a part of the finished goods**. It has several classifications such as: i) production inventories (includes raw materials, components, etc.); ii) work in progress inventories; iii) finished goods inventories; iv) MRO inventories (stands for maintenance, repair and operating. This is the inventory you use to support the manufacturing process); v) Miscellaneous inventories (scrap, stationary items etc.)

Indirect Inventories: Inventories are required for production and **are not a part of the finished goods**. Some example includes fuel, maintenance, oil, spare parts, coolants, tools, etc). It has several classifications such as: i) Transport Inventories (items currently under transportation, for example coal [or, petrolium oil] being transported from coal fields [or, pertolium oil fields] to a thermal plant [or, petrolium industry]); ii) Buffer Inventories (balancing the fluctuations on supply and demand, if there is a requirement of above average product then buffer inventories play a crucial role, also if there is a delay in supply then buffer inventories help to run the process smoothly); iii) Decouple Inventories (here inventory managers reserve a portion of the stock for each node of production, for example, the computer manufacturer would set aside a portion of the parts needed at each stage of building a laptop as a buffer against inventory interruptions in the operation's nodes. The decoupling inventories act as shock absorbers in case of varying work-rates, machine breakdowns or failures, etc.); iv) Seasonal Inventories (inventories with seasonal demands e.g. demand of woollen textiles in winter, air conditioners in summer, etc. Seasonal inventories has to be maintained to smooth production under high seasonal demand); v) Anticipation/ Provision Inventories (used to meet the anticipated demand e.g. purchasing of crackers well before Diwali, cricket match. Storing of raw materials for some expected strike, etc.); vi) Lot Size Inventories (used to take advantages of discounts for purchase of large quantities. It includes: price discounts, transportation costs discount, handling costs discounts.)

Question 4.2. *Discuss various types of costs associated with inventory control models.*
★ *Inventory costs are mainly four types: Purchasing Cost, Holding Cost, Setup Cost and Shortage Cost. We will briefly discuss these costs as follows:*

1. *Purchase Costs: Purchase cost is the price that is paid for purchasing an item. It may be constant per unit or may vary with the quantity purchased. If the cost is constant, it does not affect the inventory control decision. However, the purchase cost is definitely considered when it varies as in quantity discount situations. At times the item is offered at a discount if the order size exceeds a certain amount, which is a factor in deciding how much to order.*

2. *Holding Costs: It consists of Interest costs on capital (e.g. an interest has to be paid to the bank against the capital amount), Storage costs (e.g. rent for storage and its ideal conditions i.e. heat, cold, pressure, light, etc. Note that, for the company's own warehouse the rental cost is an opportunity cost i.e. there is a possibility to use the warehouse for other purposes is compromised), Depreciation costs (e.g. this cost mainly associated with seasonal, fashion, chemical, fragile, etc. items),*

Pilferage costs (i.e. small theft performed repeatedly over a long period of time e.g. an employee stealing small amounts of office supplies from their workplace every few days), Obsolescence costs (i.e. inventory has not been sold or used for a long period of time and is not expected to be sold in the future. So the inventory at the end of its product life cycle. For example, clothing and apparel retailers have the most difficulty with obsolescence. As every season and every year clothing styles change. So the retailers can't afford to used outdated fashion due to decrease in sales. It causes season sales as every reatiler want to get rid of the inventory before it becomes obsolete and worthless to them), Handling costs (i.e. costs associated with movement of stock e.g. labour cost, fork lifter cost, cranes cost, battery truck, etc.), Record Keeping costs (i.e. costs associated with a systematic process that enables the accountability about the whole stock and its very important) and Tax-insurance costs (i.e. tax liabiliies and costs associated with insurance cover against possible loss from damage, theft, fire, business interruption etc.)

3. **Setup Costs**: *These include the fixed cost associated with placing of an order or setting up a machinery before starting production. It includes various clerical and administrative costs (e.g. costs of purchase, requisition, follow up, receiving the goods, quality control, cost of communications, salaries of persons for accounting and auditing, etc. i.e. Ordering costs). Setup costs are independent of the quantity ordered although it directly proportional to the number of orders placed.*

   ▷ *There is an inverse relationship between setup and holding cost. Higher the order quantity reduces the cost due to setup (w.r.t. a given demand) but it will block capital and related holding costs increases. Consequently, lower the order quantity decreases the block capital and related holding costs but it increases the number of ordering and the consequent setup costs. In inventory management, we usually optimize these two costs.*

4. **Shortage Costs**: *These costs are the damage due to lack of supply (i.e. backlog) for a particular demand and the cost is usually proportional to the deficit items and delay time. Shortage cost also includes the inadequacy to supply the demand at all (i.e. demand lost). It includes potential loss of income, cancelled orders, lost sales, profit and the more subjective cost of loss in customer's goodwill.*

Moreover, if the purchase cost is constant and independent of the quantity purchased, it is not considered in the model else it is considered in the model i.e., usually **Inventory Cost = Purchase cost + Holding cost + Setup cost + Shortage cost**. □

4.1.3 Inventory Control Problems

■ Inventory modeling deals with detennining the level of a commodity that a business must maintain to ensure smooth operation.

► The basis for the decision is a model that balances the cost of capital resulting from

holding too much inventory against the penalty cost resulting from inventory shortage.
► The principal factor affecting the solution is the nature of the demand: deterministic or probabilistic. In real life, demand is usually probabilistic, but in some cases the simpler deterministic approximation may be acceptable.
► The complexity of the inventory problem does not allow the development of a general model that covers all possible situations. So we will use different model to represent various situations.

■ Usually an inventory model seeks three basic results: i) When to order ii) How much to order and iii) How much safety stock.
► i) When to order: This is related to the lead time i.e. the time interval between the placement of an order for an item and its receipt in stock. There should be enough stock for each item so that customers' orders can be reasonably met from this stock until refill and this stock level is known as reorder level.
► ii) How much to order: This is related to both the holding and setup costs. An inventory cost model balances these two costs.
► iii) How much safety stock: This is related to both the over-stock and out-stock costs. An inventory cost model balances these two costs.

■ Importance of Demand: The inventory control policy of an organisation depends upon the demand characteristics. The demand for an item may be deterministic or probabilistic. Also demand may or, may not vary over the time. The forecasting of demand is very important and usually we assume four types of demand in inventory modelling:

1. Deterministic and constant (static) with time.
2. Deterministic and variable (dynamic) with time.
3. Probabilistic and stationary over time (i.e. statistical properties are constant over time).
4. Probabilistic and nonstationary over time (i.e. statistical properties changes over time).

Now to develop an inventory model the demand type 1 is the simplest after that the complexity increases respectively for type 2, 3 and 4. However, in practical situation the order reverses i.e. 4^{th} is most likely and 1^{st} is less likely. So in modelling process one aim for a balance between model simplicity (not too simple to explain a practical situation inefficiently) and model accuracy (not too complex to handle analytically).
► To choose the demand type for inventory modelling we follow certain rules. Suppose we have a monthly consumption data for a product for 10 years (say 2010 – 2020) i.e. 120 data points. Now to check the demand for a particular month we will find mean i.e. m and coefficient of variation i.e. $C.O.V. = \frac{SD}{Mean} \times 100\%$. So there will be 12 m and C.O.V. values w.r.t. each month by using 10 year data.
▷ We will use Type 1 demand if m is approximately constant and C.O.V. is less than 20%.
▷ We will use Type 2 demand if m is not approximately constant and C.O.V. is less than 20%.
▷ We will use Type 3 demand if m and C.O.V. are approximately constant and C.O.V

greater than 20%.

▷ We will use Type 4 demand if m and C.O.V. are not approximately constant and C.O.V greater than 20%.

[Do It Yourself] 4.1. *Suppose a hospital ordered first aid kit from a manufacturer during each month for* 8 *years as follows: If the manufacturer want to forecast the future demand*

Monthly Number of First Aid Kit (in thousands) Consumption for 8 Years												
Year	Jan	Feb	Mar	Apr	May	June	July	Aug	Sep	Oct	Nov	Dec
2015	210	165	106	130	350	205	237	391	374	331	362	254
2016	281	161	109	166	320	246	240	355	382	355	354	264
2017	275	148	116	172	287	231	222	353	383	356	386	300
2018	290	178	137	157	310	234	221	352	392	332	383	287
2019	200	159	114	109	301	237	244	389	384	352	381	273
2020	178	169	138	156	345	231	230	346	410	349	367	309
2021	276	146	107	174	289	201	234	353	389	335	375	247
2022	266	155	128	184	298	202	232	323	407	347	381	276

Table 4.1: Data for Demand Type

using inventory modelling, which type of demand he will consider and why?

Example 4.1. *Briefly write down various components of inventory models.*

★ *Inventory models can be identified with lots of components. Here we are briefly discussing its components below:*

► *A)* *Demand Rate*: *It has mainly four types*

1. *Deterministic and constant (static) with time.*
2. *Deterministic and variable (dynamic) with time.*
3. *Probabilistic and stationary over time (i.e. statistical properties are constant over time).*
4. *Probabilistic and nonstationary over time (i.e. statistical properties changes over time).*

► *B)* *Replenishment Type*: *It has mainly two types*

1. *Instant replenishment (rate infinite) i.e. lead time zero.*
2. *Non-instant replenishment (rate finite) i.e. lead time non-zero. So stock gradually increases.*

► *C)* *Shortage Type*: *It has mainly two types*

1. *Shortage not allowed i.e. inventory level never goes below zero.*
2. *Shortage allowed i.e. inventory level may goes below zero.*

► *D)* *Quantity Discount*: *It has mainly two types*

1. *Quantity discount allowed i.e. inventory unit cost vary with respect to quantity.*
2. *Quantity discount not allowed i.e. inventory unit cost does not vary with respect to quantity.*

► *E)* Cost Related: *Based on the above components mainly four types of cost used in inventory models:*

1. *Purchase cost.*
2. *Setup cost.*
3. *Holding cost.*
4. *Shortage cost.* □

4.2 Deterministic Inventory Models

◘ Since a general inventory model is very complex and may not be possible to solve analytically. So we will use different models to study various scenerio.
► For Deterministic Inventory Models two broad categories are there: i) Demand rate is constant and ii) Demand rate is variable.
► The most common inventory situation faced by manufacturers, retailers, and wholesalers is that stock levels are depleted over time and then are replenished by the arrival of a batch of new units. A simple model representing this situation is the economic order quantity (EOQ) model.
◘ Some Notations: D = Demand rate (units/time), I = No. of inventories, Q = Order quantity (units) at each ordering cycle.

► Cost Notations: C_1 = Purchase cost per unit , C_2 = Ordering cost (per order), C_3 = Holding cost per inventory per time, C_4 = Shortage cost.

► Time Notations: t = Ordering cycle length (time), T = Total time to hold the inventory.

◘ Some Terms: Lead Time = Time between the placement of an order and its receiptence, Reorder Point = Inventory level at which the order is placed.

4.2.1 EOQ Model - I [A] (Shortage Not Allowed Type)

♦ [A]: Demand Rate Uniform, Instant Replenishment.

An organisation must control its inventory for smooth performance. A basic inventory situation focouses on stock levels that are decreasing with time and stock refill with new units. A simple represenation of this situtaion can be explained through economic order quantity (EOQ) model. The EOQ models are based on some elementary assumptions.

◘ Assumptions: The main assumptions of classical EOQ models are

1. A known constant demand rate (D).
2. Instantaneous order (quantity Q) replenishment (restoration of a stock) i.e. replenish rate infinite. In other words, lead time is 0 i.e. instantaneous order placed and received.
3. No shortages are allowed i.e. Inventory cost = Purchase + Setup + Holding. □

The objective of EOQ model is to determine the frequency of order i.e. when and by how much (i.e. number of unit) we should order the inventory so that it minimizes the sum of these costs per unit time.

■ Objective: The main target of classical EOQ models are to determine

1. The frequency of order i.e. optimal ordering cycle length (t).
2. The number of unit in each order i.e. optimum order quantity units ($\hat{Q}$) at each ordering cycle. □

Suppose units of the product under consideration are assumed to be withdrawn from inventory continuously at a known constant rate D per unit time (Assumption 1). It is further assumed that inventory is replenished when needed by ordering a batch of fixed size (Q units), where all Q units arrive simultaneously (Assumption 2) at the desired time (Assumption 3). The time between consecutive replenishments of inventory is referred to as a cycle (t). For the basic EOQ model, the only costs to be considered are: C_1 = Purchase cost per unit , C_2 = Ordering cost (per order), C_3 = Holding cost per inventory per time. For the fixed demand rate, shortages can be avoided by replenishing inventory each time the inventory level drops to zero (See Figure 4.1), and this also will minimize the holding cost. We assume the cost of the items remains constant over time i.e. no quantity discounts.

From the schematic diagram (Figure 4.1) it is observed that, x-axis represents time and y-axis represents inventory level. At the begining, we start fixed Q units (see the line OB) of inventory and as time goes, the inventory decreases with a constant demand rate D i.e. at an instance t_1 the $(Q - D \times t_1)$ units of inventory present in the store. Also the inventory units goes to zero at the end of the cycle (i.e. after time t) it implies $Q - D \times t = 0 \Rightarrow Q = Dt$. So the total holding inventory units during the time t i.e. the cycle length OA is = the area of the triangle OAB = $\frac{Qt}{2}$.

At the point A i.e. after the first cycle having length t, the inventory goes to zero and instant restoration the stock with Q units (see the line AB_1). The process continues upto n orders.

So, the various costs for the first cycle (i.e. per cycle) are:
Purchase cost = Item units × Cost per item = QC_1, Setup Cost = C_2, Holding Cost = Inventory units × Cost per item = $\frac{QtC_3}{2}$.
So the total cost per cycle = $QC_1 + C_2 + \frac{QtC_3}{2} = DC_1t + C_2 + \frac{DC_1C_3t^2}{2}$.
Hence, the total cost per unit time is $C = DC_1 + \frac{C_2}{t} + \frac{DC_1C_3t}{2}$.

Now we will minimize C with respect to t, $\frac{dC}{dt} = 0 \Rightarrow -\frac{C_2}{t^2} + \frac{DC_1C_3}{2} = 0 \Rightarrow \boxed{t = \sqrt{\frac{2C_2}{DC_1C_3}}}$.

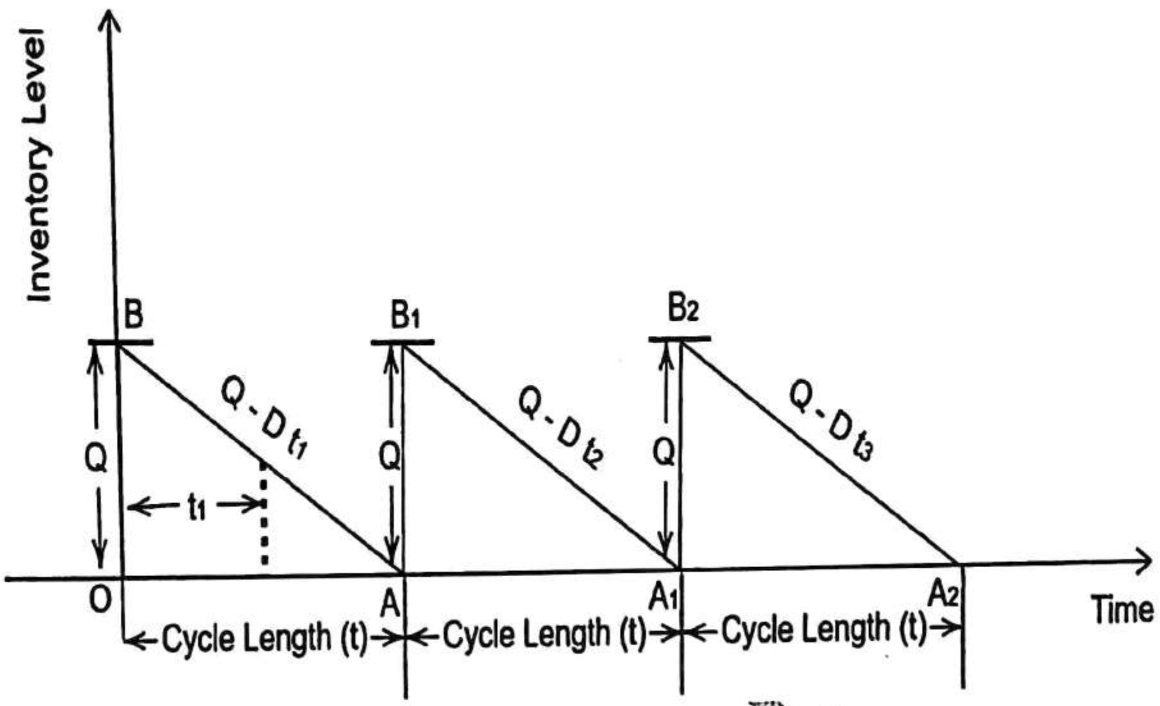

Figure 4.1: Inventory Situation with uniform demand rate and instantaneous order replenishment.

Further, $\frac{d^2C}{dt^2} > 0$ at $t = \sqrt{\frac{2C_2}{DC_1C_3}}$.

The optimal time interval of ordering or, cycle length (say t_{opt}) is $\boxed{t_{opt} = \sqrt{\frac{2C_2}{DC_1C_3}}}$.

The optimal order quantity (or, lot size) (say Q_{opt}) is $\boxed{Q_{opt} = Dt_{opt} = \sqrt{\frac{2C_2D}{C_1C_3}}}$.

The above expression is known as economic order quantity formula or, square root formula.

Example 4.2: *Write down some limitations of EOQ model.*

4.2.2 EOQ Model - I [B] (Shortage Not Allowed Type)

♦ [B]: Demand Rate Non-Uniform, Instant Replenishment.

An organisation must control its inventory for smooth performance. A basic inventory situation focouses on stock levels that are decreasing with time and stock refill with new units. A simple represenation of this situtaion can be explained through economic order quantity (EOQ) model. The EOQ models are based on some elementary assumptions.

■ Assumptions: The main assumptions of the above EOQ models are

1. A non-uniform demand rate at each cycle with total demand D_T.
2. Instantaneous order (quantity Q) replenishment (restoration of a stock) i.e. replenish rate infinite. In other words, lead time is 0 i.e. instantaneous order placed and received.
3. No shortages are allowed i.e. Inventory cost = Purchase + Setup + Holding. □

The objective of EOQ model is to determine the frequency of order i.e. when and by how much (i.e. number of unit) we should order the inventory so that it minimizes the total costs per unit time.

■ Objective: The main target of the EOQ model (I[B]) are to determine

1. The frequency of order i.e. optimal ordering cycle length (t). However, it will not applicable here as cycle lengths are different due to non-uniform demand.
2. The number of unit in each order i.e. optimum order quantity (fixed) units (Q) at each ordering cycle. □

Suppose units of the product under consideration are assumed to be withdrawn from inventory continuously at non-uniform rate $D_1, D_2, \cdots, D_n$ corresponding to the cycle of length $t_1, t_2, \cdots, t_n$ respectively per unit time (Assumption 1). The total time is $T = t_1 + \cdots + t_n$ and $nQ = D_T \Rightarrow n = \frac{D_T}{Q}$. It is further assumed that inventory is replenished when needed by ordering a batch of fixed size (Q units), where all Q units arrive simultaneously (Assumption 2) at the desired time (Assumption 3). The time between consecutive replenishments of inventory is referred to as a cycle and we assume n cycles are there i.e. there will be n orders. For the above EOQ model, the only costs to be considered are: C_1 = Purchase cost per unit, C_2 = Ordering cost (per order), C_3 = Holding cost per inventory per time. For the non-uniform demand rate, shortages can be avoided by replenishing inventory each time the inventory level drops to zero (See Figure 4.2), and this also will minimize the holding cost. We assume the cost of the items remains constant over time i.e. no quantity discounts.

From the schematic diagram (Figure 4.2) it is observed that, x-axis represents time and y-axis represents inventory level. At the begining, we start fixed Q units (see the line OB) of inventory and as time goes, the inventory decreases with a constant demand rate D_1 i.e. at an instance t the $(Q - D_1 \times t)$ units of inventory present in the store. Also the inventory units goes to zero at the end of the cycle (i.e. after time t_1) it implies $Q - D_1 \times t_1 = 0 \Rightarrow Q = D_1 t_1$. So the total holding inventory units during the time t_1 i.e. the cycle length OA is = the area of the triangle OAB = $\frac{Qt_1}{2}$.

At the point A i.e. after the first cycle having length t_1, the inventory goes to zero and instant restoration the stock with Q units (see the line AB_1). The process continues upto n orders.

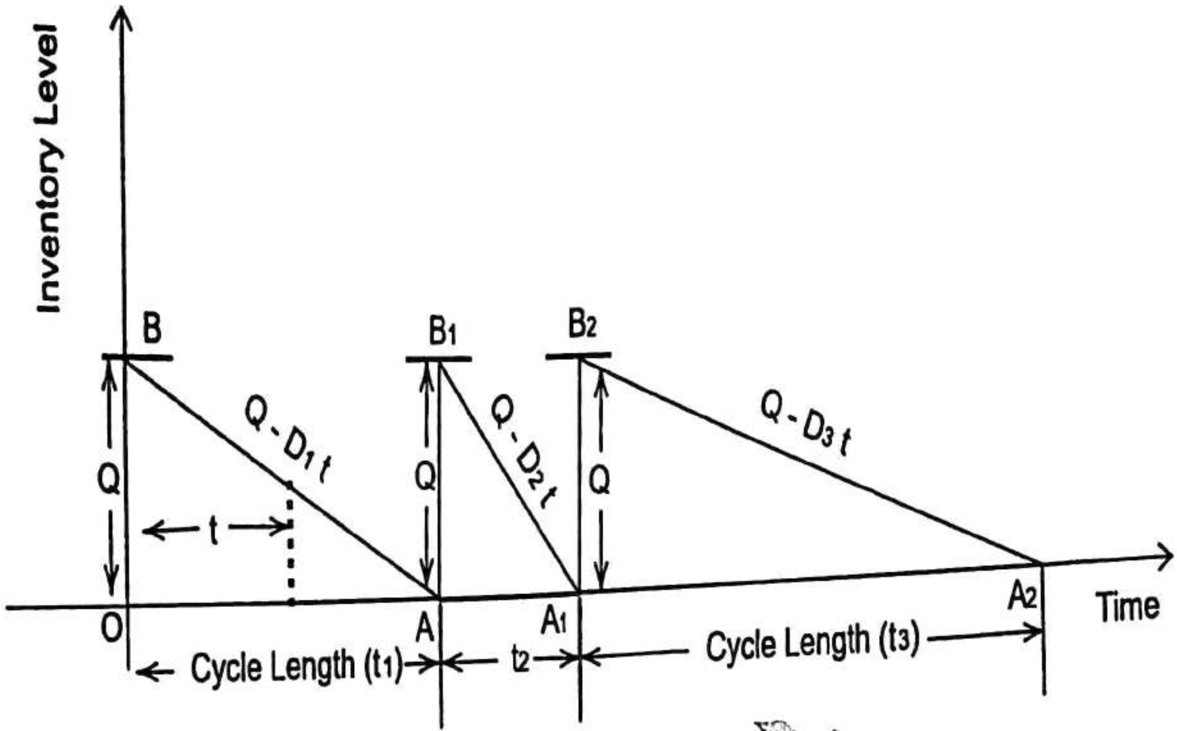

Figure 4.2: Inventory Situation with non-uniform demand rate and instantaneous order replenishment.

So, the various costs for the first cycle (i.e. per cycle) are:
Purchase cost = Item units × Cost per item = QC_1, Setup Cost = C_2, Holding Cost = Inventory units × Cost per item = $\frac{Qt_1C_3}{2}$.
So the total cost per cycle = $QC_1 + C_2 + \frac{Qt_1C_3}{2}$.
Hence, the total cost for n cycles is $C = nQC_1 + nC_2 + \frac{QC_3}{2}(t_1 + \cdots + t_n) = D_TC_1 + \frac{D_TC_2}{Q} + \frac{QC_3T}{2}$.

Now we will minimize C with respect to Q, $\frac{dC}{dQ} = 0 \Rightarrow -\frac{D_TC_2}{Q^2} + \frac{C_3T}{2} = 0 \Rightarrow \boxed{Q = \sqrt{\frac{2D_TC_2}{C_3T}}}$.

Further, $\frac{d^2C}{dQ^2} > 0$ at $Q = \sqrt{\frac{2D_TC_2}{C_3T}}$.

The optimal order quantity (or, lot size) (say Q_{opt}) is $\boxed{Q_{opt} = \sqrt{\frac{2D_TC_2}{C_3T}}}$.

The optimal cost (say C_{opt}) is $\boxed{C_{opt} = D_TC_1 + \frac{D_TC_2}{Q_{opt}} + \frac{Q_{opt}C_3T}{2} = D_TC_1 + \sqrt{2D_TC_2C_3T}}$.

4.2.3 EOQ Model - I [C] (Shortage Not Allowed Type)

♦ [C]: Demand Rate Uniform, Non Instant Replenishment.

An organisation must control its inventory for smooth performance. A basic inventory situation focouses on stock levels that are decreasing with time and stock refill with new units. A simple represenation of this situtaion can be explained through economic order quantity (EOQ) model. The EOQ models are based on some elementary assumptions.

Assumptions: The main assumptions of the above EOQ models are

1. A uniform demand rate at each cycle with demand rate D.
2. Non Instantaneous order replenishment (restoration of a stock) i.e. replenish rate finite. Here each cycle has two parts: First part starts with zero inventory and inventory increases with a specific rate (I) upto certain time (say t_1) (while demand is simultaneously acting). Second part there will be no supply of inventory, demand continues and after a certain time (say t_2) the inventory goes to zero. So each cycle has length $t = t_1 + t_2$. Same continues in all the cycle.
3. No shortages are allowed i.e. Inventory cost = Purchase + Setup + Holding. □

The objective of EOQ model is to determine the frequency of order i.e. when and by how much (i.e. number of unit) we should order the inventory so that it minimizes the total costs per unit time.

Objective: The main target of the EOQ model (I[C]) are to determine

1. The optimal cycle length (t).
2. The number of unit in each order i.e. optimum order quantity units (Q) at each cycle. □

Suppose units of the product under consideration are assumed to be withdrawn from inventory continuously at an uniform rate D at each cycle of length t per unit time (Assumption 1). The total item produced is $Q = D \times t$. It is further assumed that in each cycle lenth t, inventory is replenished with a rate I per time ($> D$ as no shortage allowed by Assumption 3) for time t_1, and the remaining inventory units (say Q_r) goes to zero at time t_2 with demand rate D such that $t = t_1 + t_2$. The time between consecutive replenishments of inventory is referred to as a cycle and we assume n cycles are there. For the above EOQ model, the only costs to be considered are: C_1 = Purchase cost per unit, C_2 = Ordering cost (per order), C_3 = Holding cost per inventory per time. For the uniform demand rate, shortages can be avoided by replenishing inventory each time the inventory level drops to zero (See Figure 4.3), and this also will minimize the holding cost. We assume the cost of the items remains constant over time i.e. no quantity discounts.

From the schematic diagram (Figure 4.3) it is observed that, x-axis represents time and y-axis represents inventory level. At the begining, we start from zero units (see the point O) of inventory with replenishment rate I and demand rate D per unit time ($I > D$) upto time t_1. At the point E, the inventory present is $BE = Q_r$ units. After t_1 unit of time, there will be no inventory replenishment, only demand with rate D, an instance t'' the $(Q_r - D \times t'')$ units of inventory present in the store. Also the inventory units goes to zero at the end of the cycle (i.e. after time t_2) it implies $Q_r - D \times t_2 = 0 \Rightarrow Q_r = Dt_2$. Again, at point B, $(I - D)t_1 = Q_r$ i.e. $\frac{I-D}{D} = \frac{t_2}{t_1} \Rightarrow \frac{I}{D} = \frac{t}{t_1} \Rightarrow t_1 = \frac{Dt}{I} \Rightarrow \boxed{Q_r = \frac{(I-D)Dt}{I}}$. So the total holding inventory units during the time $t = t_1 + t_2$ i.e. the cycle length OA

is = the area of the triangle OAB = $\frac{Q_r t}{2}$.

At the point A i.e. after the first cycle having length t, the inventory goes to zero and inventory replenishment with rate I upto AE_1 (demand acts simultaneously) and then inventory decreases due to demand upto E_1A_1. The process continues upto n orders.

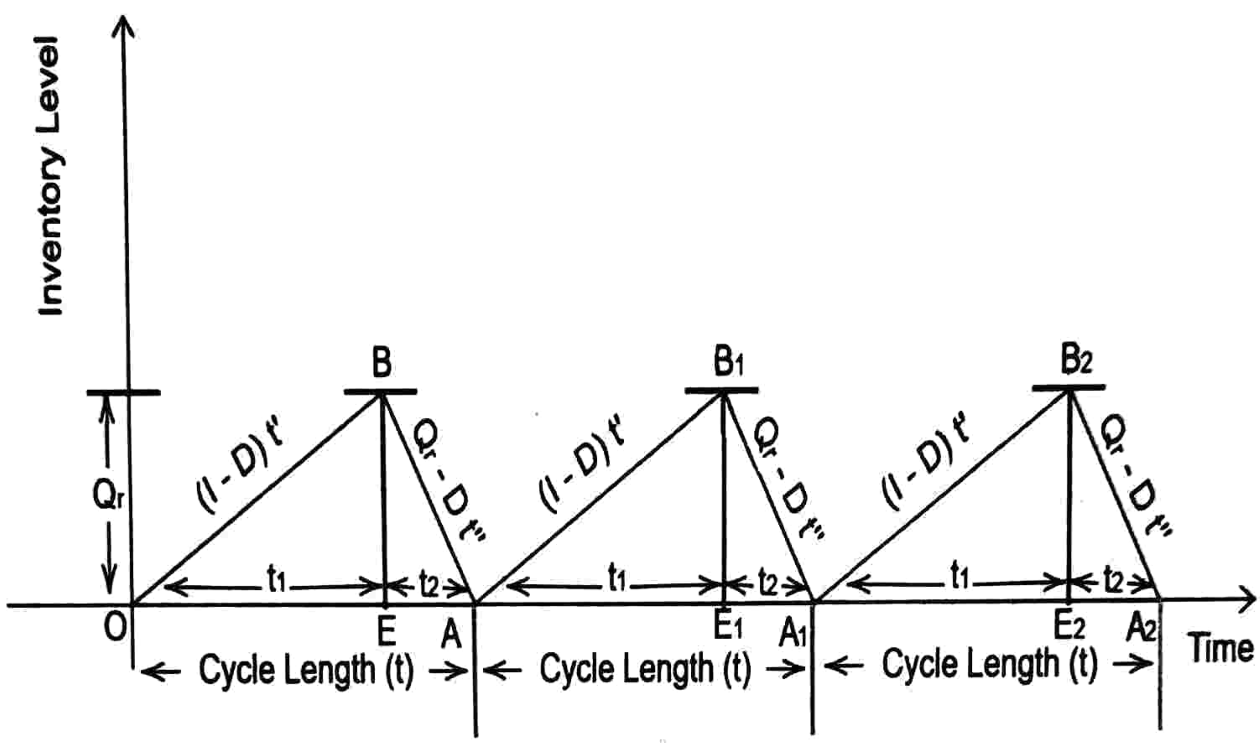

Figure 4.3: Inventory Situation with uniform demand rate and non-instantaneous order replenishment.

So, the various costs for the first cycle (i.e. per cycle) are:
Purchase cost = Item units × Cost per item = DtC_1, Setup Cost = C_2, Holding Cost = Inventory units × Cost per item = $\frac{Q_r t C_3}{2}$.
So the total cost per cycle = $DtC_1 + C_2 + \frac{Q_r t C_3}{2} = DtC_1 + C_2 + \frac{(I-D)Dt^2C_3}{2I}$.
Hence, the total cost per cycle per unit time is $C = DC_1 + \frac{C_2}{t} + \frac{(I-D)DtC_3}{2I}$.
Now we will minimize C with respect to t, $\frac{dC}{dt} = 0 \Rightarrow -\frac{C_2}{t^2} + \frac{(I-D)DC_3}{2I} = 0 \Rightarrow \boxed{t = \sqrt{\frac{2IC_2}{(I-D)DC_3}}}$.

Further, $\frac{d^2C}{dt^2} > 0$ at $t = \sqrt{\frac{2IC_2}{(I-D)DC_3}}$.

The optimal cycle length (say t_{opt}) is $\boxed{t_{opt} = \sqrt{\frac{2IC_2}{(I-D)DC_3}}}$.

The optimal order quantity (or, lot size) (say Q_{opt}) is $\boxed{Q_{opt} = Dt_{opt} = \sqrt{\frac{2IDC_2}{(I-D)C_3}}}$.

The optimal cost per cycle (say C_{opt}) is $\boxed{C_{opt} = Dt_{opt}C_1 + C_2 + \frac{(I-D)Dt_{opt}^2C_3}{2I}}$.

4.2.4 EOQ Model - II [A] (Shortage Allowed Type)

♦ [A]: Demand Rate Uniform, Instant Replenishment, Shortage Allowed.

An organisation must control its inventory for smooth performance. A basic inventory situation focouses on stock levels that are decreasing with time and stock refill with new units. A simple represenation of this situtaion can be explained through economic order quantity (EOQ) model. The EOQ models are based on some elementary assumptions.

▣ Assumptions: The main assumptions of the above EOQ models are

1. A uniform demand rate at each cycle with demand rate D.
2. Instantaneous order (quantity Q) replenishment (restoration of a stock) i.e. replenish rate infinite. In other words, lead time is 0 i.e. instantaneous order placed and received.
3. Shortages are allowed i.e. Inventory cost = Purchase + Setup + Holding + Shortage. Now, the total cycle lenth t can be split into two parts: t_1 i.e. after which inventory remains zero and t_2 i.e. upto that time there is shortage of inventory. In that t_2 time period, the demand accumulated and after t_2 time Q units of replenishment inventory instantly arrives. So $t = t_1 + t_2$ in t_1 part there will be holding cost and no shortage cost, whereas in t_2 part there will be shortage cost and no holding cost. □

The objective of EOQ model is to determine the frequency of order i.e. when and by how much (i.e. number of unit) we should order the inventory so that it minimizes the total costs per unit time.

▣ Objective: The main target of the EOQ model (II[A]) are to determine

1. The optimal cycle length (t).
2. The number of unit in each order i.e. optimum order quantity units (Q) at each cycle.
3. Also the optimum shortage unit in each cycle. □

Suppose units of the product under consideration are assumed to be withdrawn from inventory continuously at an uniform rate D at each cycle of length t per unit time (Assumption 1). The total item ordered is $Q = D \times t$. It is further assumed that in each cycle lenth t, inventory is replenished instantly with Q units (Assumption 2). With a demand rate D per time for time t_1 the inventory goes to zero, and for time t_2 with demand rate D there is a shortage of inventory (as shortage allowed by Assumption 3) such that $t = t_1 + t_2$. The time between consecutive replenishments of inventory is referred to as a cycle and we assume n cycles are there. For the above EOQ model, the only costs to be considered are: C_1 = Purchase cost per unit, C_2 = Ordering cost (per order), C_3 = Holding cost per inventory per time, C_4 = Shortage cost per inventory per time. For the uniform demand rate with shortages usually inventory replenishing each time after there is some shortage upto a certain time (See Figure 4.4), and this also will lower the holding cost. We assume the cost of the items remains constant over time i.e. no quantity discounts.

From the schematic diagram (Figure 4.4) it is observed that, x-axis represents time and y-axis represents inventory level. At the begining, we start from Q' units (see the line OB) with demand rate D per unit time, at time t_1 the Q' unit reduces to zero (at point A). So at the point A, $Q' = Dt_1$. After t_1 unit of time, there will be no inventory, only demand with rate D, so a shortage continue for the time t_2. At the point F, Q unit of inventory (i.e. B_1E) instantly added in the system. So, at point F, $Q - Dt_2 = Q' \Rightarrow Q = Dt$. So the total holding inventory units during the time t_1 is the area of triangle $OAB = \frac{Q't_1}{2}$, inventory shortage during time t_2 is the area of the triangle $AEF = \frac{(Q-Q')t_2}{2}$ and the cycle length OF is $= t = t_1 + t_2$.

▶ Here consider similar triangles EB_1E_1 and FB_1A_1, then we have:

$\frac{B_1F}{B_1E} = \frac{FA_1}{EE_1} \Rightarrow \frac{Q'}{Q} = \frac{t_1}{t} \Rightarrow \boxed{t_1 = \frac{Q't}{Q} = \frac{Q'}{D}}$,

Again $\frac{A_1F_1}{FF_1} = \frac{F_1E_1}{B_1E} \Rightarrow \frac{t_2}{t} = \frac{Q-Q'}{Q} \Rightarrow \boxed{t_2 = \frac{(Q-Q')t}{Q}}$.

At the point F i.e. after the first cycle having length t, the inventory goes to negative value (i.e. deficit EF) and inventory replenishment instantly Q units (i.e. EB_1). The process continues upto n orders.

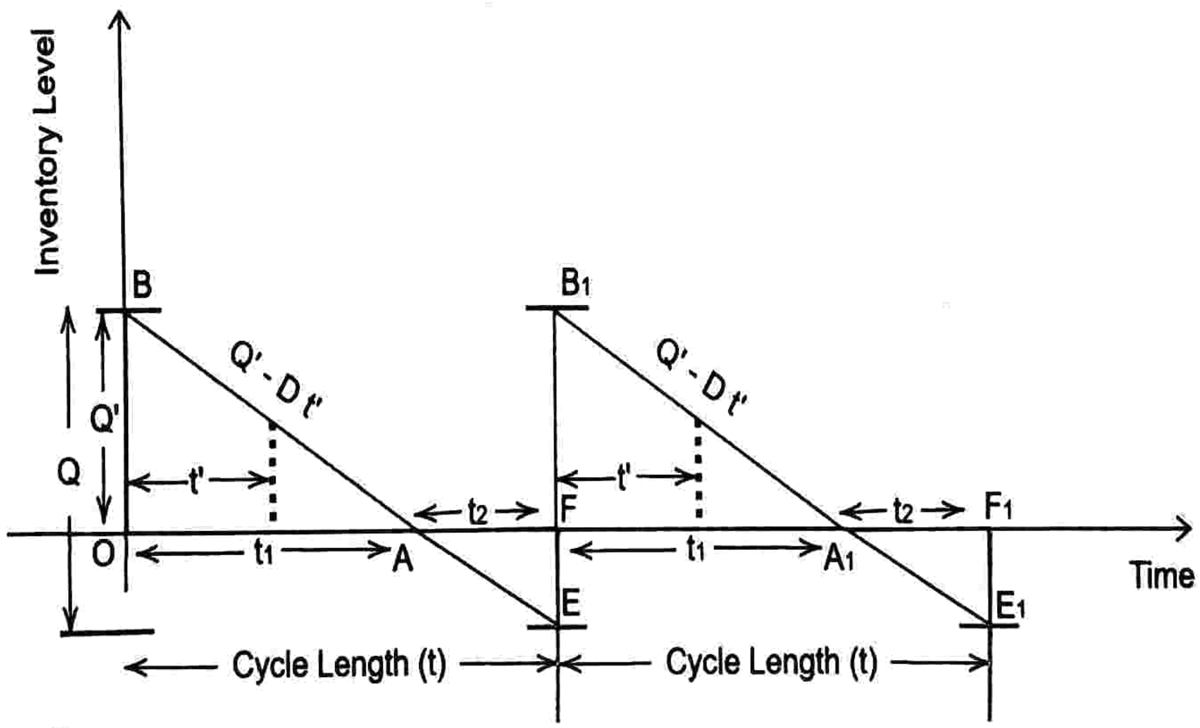

Figure 4.4: Inventory situation with uniform demand rate, instantaneous order replenishment and shortage allowed.

So, the various costs for the first cycle is:
Purchase cost = Item units × Cost per item = QC_1, Setup Cost = C_2, Holding Cost =

Inventory units × Cost per item = $\frac{Q't_1C_3}{2}$, Shortage Cost = Inventory shortage units × Shortage cost per item = $\frac{(Q-Q')t_2C_4}{2}$.
So the total cost per cycle = $QC_1 + C_2 + \frac{Q't_1C_3}{2} + \frac{(Q-Q')t_2C_4}{2} = DtC_1 + C_2 + \frac{Q'^2tC_3}{2Q} + \frac{(Q-Q')^2tC_4}{2Q}$.
Hence, the total cost per cycle per unit time is $C = DC_1 + \frac{C_2}{t} + \frac{Q'^2C_3}{2Q} + \frac{(Q-Q')^2C_4}{2Q} = DC_1 + \frac{C_2D}{Q} + \frac{Q'^2C_3}{2Q} + \frac{(Q-Q')^2C_4}{2Q}$.
Now we will minimize C with respect to Q and Q', $\frac{\partial C}{\partial Q} = 0$ and $\frac{\partial C}{\partial Q'} = 0$.
Now, $\frac{\partial C}{\partial Q} = 0 \Rightarrow -\frac{C_2D}{Q^2} - \frac{Q'^2C_3}{2Q^2} + \frac{C_4}{2} \times \frac{Q.2(Q-Q')-(Q-Q')^2}{Q^2} = 0 \Rightarrow 2C_2D + Q'^2C_3 + C_4(Q'^2 - Q^2) = 0 \quad \cdots \quad (1)$.
$\frac{\partial C}{\partial Q'} = 0 \Rightarrow \frac{2Q'C_3}{2Q} - \frac{C_4}{2} \times \frac{2(Q-Q')}{Q} = 0 \Rightarrow Q'C_3 = (Q-Q')C_4 \Rightarrow QC_4 = Q'(C_3+C_4) \quad \cdots \quad (2)$
.
Solving (1) and (2), we have $(Q^2-Q'^2)C_4 - Q'^2C_3 = 2C_2D \Rightarrow Q'^2\{[\frac{(C_3+C_4)^2}{C_4^2} - 1]C_4 - C_3\} = 2C_2D \Rightarrow Q'^2\{[\frac{C_3^2+2C_3C_4}{C_4}] - C_3\} = 2C_2D \Rightarrow Q'^2[\frac{C_3^2+C_3C_4}{C_4}] = 2C_2D \Rightarrow$ $\boxed{Q' = \sqrt{\frac{2C_2D}{C_3}}\sqrt{\frac{C_4}{C_3+C_4}}}$

So by (2), $\boxed{Q = \sqrt{\frac{2C_2D}{C_3}}\sqrt{\frac{C_3+C_4}{C_4}}}$.

Further, $\frac{\partial^2 C}{\partial Q^2}\frac{\partial^2 C}{\partial Q'^2} - \frac{\partial^2 C}{\partial Q \partial Q'} > 0$ at $Q = \sqrt{\frac{2C_2D}{C_3}}\sqrt{\frac{C_3+C_4}{C_4}}$, $Q' = \sqrt{\frac{2C_2D}{C_3}}\sqrt{\frac{C_4}{C_3+C_4}}$.

The optimal order quantity (or, lot size) (say Q_{opt}) is $\boxed{Q_{opt} = \sqrt{\frac{2C_2D}{C_3}}\sqrt{\frac{C_3+C_4}{C_4}}}$.

The optimal shortage (say $S_{opt} = Q_{opt} - Q'_{opt}$) is $\boxed{S_{opt} = \sqrt{\frac{2C_2D}{C_3}}\sqrt{\frac{C_3^2}{C_4(C_3+C_4)}} = \sqrt{\frac{2C_2D}{C_4}}\sqrt{\frac{C_3}{C_3+C_4}}}$.

The optimal cycle length (say t_{opt}) is $\boxed{t_{opt} = \frac{Q_{opt}}{D} = \sqrt{\frac{2C_2}{C_3D}}\sqrt{\frac{C_3+C_4}{C_4}}}$.

The optimal cost per cycle (say C_{opt}) is $\boxed{C_{opt} = Dt_{opt}C_1 + C_2 + \frac{Q'^2_{opt}t_{opt}C_3}{2Q_{opt}} + \frac{S^2_{opt}t_{opt}C_4}{2Q_{opt}}}$.

4.2.5 EOQ Model - II [B] (Shortage Allowed Type)

♦ [B]: Demand Rate Uniform, Instant Replenishment, Shortage Allowed, Fixed cycle length.

An organisation must control its inventory for smooth performance. A basic inventory situation focouses on stock levels that are decreasing with time and stock refill with new units. A simple represenation of this situtaion can be explained through economic order quantity (EOQ) model. The EOQ models are based on some elementary assumptions.

◘ Assumptions: The main assumptions of the above EOQ models are

1. A uniform demand rate at each cycle with demand rate D.
2. Instantaneous order (quantity Q) replenishment (restoration of a stock) i.e. replen-

ish rate infinite. In other words, lead time is 0 i.e. instantaneous order placed and received.

3. Shortages are allowed i.e. Inventory cost = Purchase + Setup + Holding + Shortage. Now, the total cycle lenth t can be split into two parts: t_1 i.e. after which inventory remains zero and t_2 i.e. upto that time there is shortage of inventory. In that t_2 time period, the demand accumulated and after t_2 time Q units of replenishment inventory instantly arrives. So $t = t_1 + t_2$ in t_1 part there will be holding cost and no shortage cost, whereas in t_2 part there will be shortage cost and no holding cost.
4. Fixed cycle length i.e. t is fixed $\Rightarrow Q$ is fixed as $Q = D \times t$. □

The objective of EOQ model is to determine the frequency of order i.e. when and by how much (i.e. number of unit) we should order the inventory so that it minimizes the total costs per unit time.

Objective: The main target of the EOQ model (II[B]) are to determine

1. The optimal cycle length (t). However, it is not applicable as t is fixed.
2. The number of unit in each order i.e. optimum order quantity units (Q) at each cycle. However, it is not applicable as Q is fixed.
3. Also the optimum shortage unit in each cycle. □

Suppose units of the product under consideration are assumed to be withdrawn from inventory continuously at an uniform rate D at each cycle of length t per unit time (Assumption 1). The total item ordered is $\boxed{Q = D \times t}$. It is further assumed that in each fixed cycle lenth t (Assumption 4), inventory is replenished instantly with Q units (Assumption 2). With a demand rate D per time for time t_1 the inventory goes to zero, and for time t_2 with demand rate D there is a shortage of inventory (as shortage allowed by Assumption 3) such that $t = t_1 + t_2$. The time between consecutive replenishments of inventory is referred to as a cycle and we assume n cycles are there. For the above EOQ model, the only costs to be considered are: C_1 = Purchase cost per unit, C_2 = Ordering cost (per order), C_3 = Holding cost per inventory per time, C_4 = Shortage cost per inventory per time. For the uniform demand rate with shortages usually inventory replenishing each time after there is some shortage upto a certain time (See Figure 4.5), and this also will lower the holding cost. We assume the cost of the items remains constant over time i.e. no quantity discounts.

From the schematic diagram (Figure 4.5) it is observed that, x-axis represents time and y-axis represents inventory level. At the begining, we start from Q' units (see the line OB) with demand rate D per unit time, at time t_1 the Q' unit reduces to zero (at point A). So at the point A, $Q' = Dt_1$. After t_1 unit of time, there will be no inventory, only demand with rate D, so a shortage continue for the time t_2. At the point F, Q unit of inventory (i.e. B_1E) instantly added in the system. So, at point F, $Q - Dt_2 = Q' \Rightarrow Q = Dt$. So the total holding inventory units during the time t_1 is the area of triangle $OAB = \frac{Q't_1}{2}$, inventory shortage during time t_2 is the area of the triangle $AEF = \frac{(Q-Q')t_2}{2}$ and the cycle length OF is $= t = t_1 + t_2$ is fixed.

▶ Here consider similar triangles EB_1E_1 and FB_1A_1, then we have:

$\frac{B_1F}{B_1E} = \frac{FA_1}{EE_1} \Rightarrow \frac{Q'}{Q} = \frac{t_1}{t} \Rightarrow \boxed{t_1 = \frac{Q't}{Q} = \frac{Q'}{D}}$,

Again $\frac{A_1F_1}{FF_1} = \frac{F_1E_1}{B_1E} \Rightarrow \frac{t_2}{t} = \frac{Q-Q'}{Q} \Rightarrow \boxed{t_2 = \frac{(Q-Q')t}{Q}}$.

At the point F i.e. after the first cycle having length t, the inventory goes to negative value (i.e. deficit EF) and inventory replenishment instantly Q units (i.e. EB_1). The process continues upto n orders.

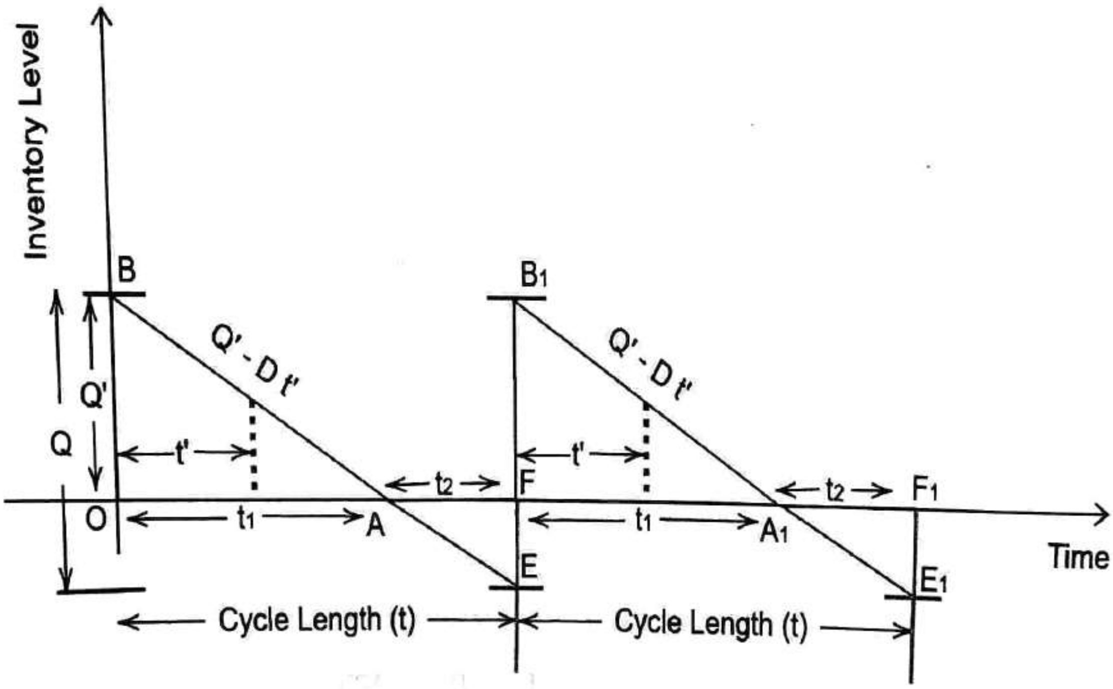

Figure 4.5: Inventory situation with uniform demand rate, instantaneous order replenishment and shortage allowed. Fixed cycle length.

So, the various costs for the first cycle is:
Purchase cost = Item units × Cost per item = QC_1, Setup Cost = C_2, Holding Cost = Inventory units × Cost per item = $\frac{Q't_1C_3}{2}$, Shortage Cost = Inventory shortage units × Shortage cost per item = $\frac{(Q-Q')t_2C_4}{2}$.
So the total cost per cycle = $QC_1 + C_2 + \frac{Q't_1C_3}{2} + \frac{(Q-Q')t_2C_4}{2} = DtC_1 + C_2 + \frac{Q'^2tC_3}{2Q} + \frac{(Q-Q')^2tC_4}{2Q}$,
Hence, the total cost per cycle per unit time is $C = DC_1 + \frac{C_2}{t} + \frac{Q'^2C_3}{2Q} + \frac{(Q-Q')^2C_4}{2Q} = DC_1 + \frac{C_2D}{Q} + \frac{Q'^2C_3}{2Q} + \frac{(Q-Q')^2C_4}{2Q}$.
Since, t is fixed $\Rightarrow Q$ is fixed. So the first two terms of C are constant.
Now we will minimize C with respect to Q', i.e. $\frac{dC}{dQ'} = 0$.
Now, $\frac{dC}{dQ'} = 0 \Rightarrow \frac{2Q'C_3}{2Q} - \frac{C_4}{2} \times \frac{2(Q-Q')}{Q} = 0 \Rightarrow Q'C_3 = (Q-Q')C_4 \Rightarrow QC_4 = Q'(C_3+C_4) \Rightarrow$

$\boxed{Q' = \frac{QC_4}{C_3+C_4}}$.

Further, $\frac{d^2C}{dQ'^2} > 0$ at $Q' = \frac{QC_4}{C_3+C_4}$.

The optimal shortage (say $S_{opt} = Q - Q'_{opt}$) is $\boxed{S_{opt} = Q - \frac{QC_4}{C_3+C_4} = \frac{QC_3}{C_3+C_4} = \frac{DtC_3}{C_3+C_4}}$.

The optimal cost per cycle (say C_{opt}) is $\boxed{C_{opt} = DtC_1 + C_2 + \frac{Q'^2_{opt}tC_3}{2Dt} + \frac{S^2_{opt}tC_4}{2Dt}}$.

4.2.6 EOQ Model - II [C] (Shortage Allowed Type)

♦ [C]: Demand Rate Uniform, Non Instant Replenishment, Shortage Allowed.

An organisation must control its inventory for smooth performance. A basic inventory situation focouses on stock levels that are decreasing with time and stock refill with new units. A simple represenation of this situtaion can be explained through economic order quantity (EOQ) model. The EOQ models are based on some elementary assumptions.

☐ **Assumptions**: The main assumptions of the above EOQ models are

1. A uniform demand rate at each cycle with demand rate D.
2. Non Instantaneous order replenishment (restoration of a stock) i.e. replenish rate finite. Here each cycle has two parts: First part starts with zero inventory and inventory increases with a specific rate (I) upto certain time (say t_1) (while demand is simultaneously acting). Second part there will be no supply of inventory, demand continues and after a certain time (say t_2) the inventory goes to zero.
3. Shortages are allowed i.e. Inventory cost = Purchase + Setup + Holding + Shortage. Now, the total cycle lenth t can be split into two parts: $t_1 + t_2$ i.e. after which inventory remains zero and $t_3 + t_4$ i.e. upto that time there is shortage of inventory. In that $t_3 + t_4$ shortage time period, upto t_3 time there is a shortage and no supply of inventory. After t_3 the supply of inventory started and upto t_4 time the inventory again goes to zero from the shortage. So $t = t_1 + t_2 + t_3 + t_4$ in $t_1 + t_2$ part there will be holding cost and no shortage cost, whereas in $t_3 + t_4$ part there will be shortage cost and no holding cost. So each cycle has length $t = t_1 + t_2 + t_3 + t_4$. Same continues in all the cycle. ☐

The objective of EOQ model is to determine the frequency of order i.e. when and by how much (i.e. number of unit) we should order the inventory so that it minimizes the total costs per unit time.

☐ **Objective**: The main target of the EOQ model (II[C]) are to determine

1. The optimal cycle length (t).
2. The number of unit in each order i.e. optimum order quantity units (Q) at each cycle.
3. Also the optimum shortage unit in each cycle. ☐

Suppose units of the product under consideration are assumed to be withdrawn from inventory continuously at an uniform rate D at each cycle of length t per unit time (Assumption 1). The total item ordered is $\boxed{Q = I \times (t_1 + t_4)}$. It is further assumed that in each cycle length t, inventory is replenished with a rate I per time ($> D$ otherwise the system always runs with shortage) for time t_1 (Assumption 2), and the remaining inventory units (say Q_r) goes to zero at time t_2 with the demand rate D. For time t_3 with demand rate D there is a shortage of inventory (as shortage allowed by Assumption 3) with no replinishment rate. Lastly, for time t_4 with demand rate D there is a shortage of inventory with replinishment rate I per time such that at the end of time t_4 the inventory units goes to zero from shortage. The time between consecutive replenishments of inventory is referred to as a cycle and we assume n cycles are there. For the above EOQ model, the only costs to be considered are: C_1 = Purchase cost per unit, C_2 = Ordering cost (per order), C_3 = Holding cost per inventory per time, C_4 = Shortage cost per inventory per time. For the uniform demand rate with shortages usually inventory replenishing with respect to time after there is some shortage upto a certain time (See Figure 4.6), and this also will lower the holding cost. We assume the cost of the items remains constant over time i.e. no quantity discounts.

From the schematic diagram (Figure 4.6) it is observed that, x-axis represents time and y-axis represents inventory level. At the begining, we start from zero units (see the point O) of inventory replenishment rate I and demand rate D per unit time ($I > D$), upto time t_1. At the point E, the inventory present is $BE = Q_r$ units. After t_1 unit of time, there will be no inventory replenishment, only demand with rate D, an instance t'' the $(Q_r - D \times t'')$ units of inventory present in the store. Also the inventory units goes to zero at the end of the cycle (i.e. after time t_2) it implies $Q_r - D \times t_2 = 0 \Rightarrow$ $\boxed{Q_r = Dt_2 \quad \cdots \quad (1)}$. Again, at point B, $\boxed{(I - D)t_1 = Q_r \quad \cdots \quad (2)}$. So the Q_r unit reduces to zero at point A. After $t_1 + t_2$ unit of time (i.e. after point A), there will be no inventory, only demand with rate D, so a shortage continue for the time t_3 with a deficit $FH = S$ units. After the point H, inventory added with rate I units per time upto t_4 time and the inventory units again goes to zero (see point G) from the shortage. So at the point A, $Q_r - Dt_2 = 0$, $\boxed{S - Dt_3 = 0 \quad \cdots \quad (3)}$. Again, at point H, $Dt_3 = (I - D)t_4$. Also at G, $\boxed{S - (I - D)t_4 = 0 \quad \cdots \quad (4)}$. From (1), (3) we have $\boxed{Q_r + S = D(t_2 + t_3) \quad \cdots \quad (5)}$, and from (2), (4) we have $\boxed{Q_r + S = (I - D)(t_1 + t_4) \quad \cdots \quad (6)}$. So from (5), (6) we have $\boxed{D(t_2 + t_3) = (I - D)(t_1 + t_4) \quad \cdots \quad (7)}$. Further, from (6) we have $\boxed{Q_r + S = \frac{I-D}{I} Q \quad \cdots \quad (8)}$.

So the total holding inventory units during the time $t_1 + t_2$ is the area of triangle $OAB = \frac{Q_r(t_1+t_2)}{2}$, inventory shortage during time $t_3 + t_4$ is the area of the triangle AHG $= \frac{S(t_3+t_4)}{2}$ and cycle length OG is $= t = t_1 + t_2 + t_3 + t_4 = (t_1 + t_4)(1 + \frac{I-D}{D})$. [using (7)] It implies $t = \frac{Q}{I} \times \frac{I}{D} = \frac{Q}{D}$ [using $Q = I(t_1 + t_4)$] i.e. $\boxed{t = \frac{Q}{D} \quad \cdots \quad (9)}$.

At the point G i.e. after the first cycle having length t, the inventory goes to zero and inventory replenishment with rate I per units per time. The process continues upto n orders.

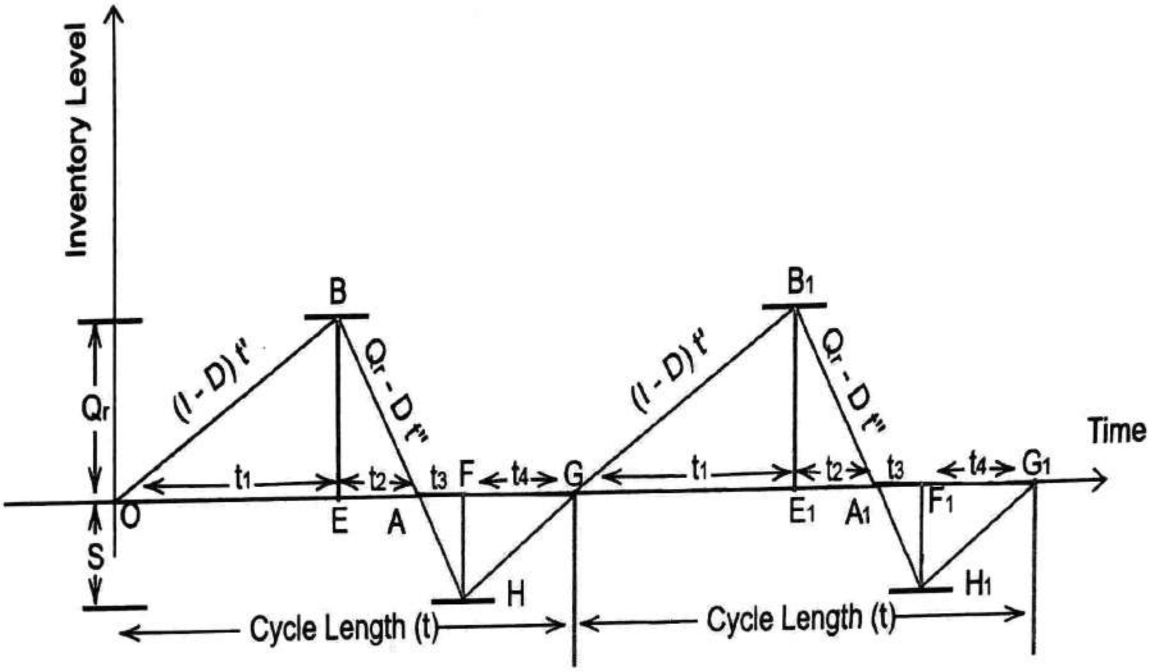

Figure 4.6: Inventory situation with uniform demand rate, non-instantaneous order replenishment and shortage allowed.

So, the various costs for the first cycle is:
Purchase cost = Item units × Cost per item = QC_1, Setup Cost = C_2, Holding Cost = Inventory units × Cost per item = $\frac{Q_r(t_1+t_2)C_3}{2}$, Shortage Cost = Inventory shortage units × Shortage cost per item = $\frac{S(t_3+t_4)C_4}{2}$.
So the total cost per cycle is $C = QC_1 + C_2 + \frac{Q_r(t_1+t_2)C_3}{2} + \frac{S(t_3+t_4)C_4}{2} = DtC_1 + C_2 + \frac{Q_rC_3}{2}(\frac{Q_r}{I-D} + \frac{Q_r}{D}) + \frac{SC_4}{2}(\frac{S}{D} + \frac{S}{I-D})$ [Using (1)-(4),(9)].
It implies, $C = DtC_1 + C_2 + \frac{Q_r^2C_3}{2}\frac{I}{D(I-D)} + \frac{S^2C_4}{2}\frac{I}{D(I-D)}$

Hence, the total cost per cycle per unit time is $C = DC_1 + \frac{C_2}{t} + \frac{Q_r^2C_3}{2t}\frac{I}{D(I-D)} + \frac{S^2C_4}{2t}\frac{I}{D(I-D)}$. Now there is relation (8) between Q and Q_r so we will replace Q_r by Q. We also replace t using (9), so we get

Therefore, $$\boxed{C = DC_1 + \frac{DC_2}{Q} + \frac{[\frac{I-D}{I}Q-S]^2C_3}{2Q}\frac{I}{(I-D)} + \frac{S^2C_4}{2Q}\frac{I}{(I-D)}}$$

Now C has two unknown components S and Q. So we will minimize C with respect to S and Q, i.e. $\frac{\partial C}{dS} = 0$ and $\frac{\partial C}{dS} = 0$.
Now, $\frac{\partial C}{\partial S} = 0 \Rightarrow -\frac{2[\frac{I-D}{I}Q-S]C_3}{2Q}\frac{I}{(I-D)} + \frac{2SC_4}{2Q}\frac{I}{(I-D)} = 0 \Rightarrow [\frac{I-D}{I}Q-S]C_3 = SC_4 \Rightarrow \frac{I-D}{I}Q = \frac{C_3+C_4}{C_3}S \Rightarrow \frac{S}{Q} = \frac{C_3}{C_3+C_4}\frac{I-D}{I} \quad \cdots \quad (10)$.
$\frac{\partial C}{\partial Q} = 0 \Rightarrow -\frac{DC_2}{Q^2} + [(\frac{I-D}{I})^2 - \frac{S^2}{Q^2}]\frac{C_3 I}{2(I-D)} - \frac{S^2C_4}{2Q^2}\frac{I}{(I-D)} = 0 \Rightarrow \frac{DC_2}{Q^2} = C_3\frac{I-D}{2I} - \frac{I}{2(I-D)} \times$

$\frac{S^2}{Q^2}(C_3 + C_4) \Rightarrow \frac{DC_2}{Q^2} = C_3\frac{I-D}{2I} - \frac{I}{2(I-D)} \times \frac{C_3^2}{(C_3+C_4)^2}\frac{(I-D)^2}{I^2}(C_3 + C_4) \Rightarrow \frac{DC_2}{Q^2} = C_3\frac{I-D}{2I} - \frac{I-D}{2I} \times \frac{C_3^2}{C_3+C_4} \Rightarrow \frac{DC_2}{Q^2} = \frac{I-D}{2I} \times \frac{C_3C_4}{C_3+C_4} \Rightarrow \boxed{Q = \sqrt{2C_2\frac{C_3+C_4}{C_3C_4}} \times \sqrt{\frac{DI}{I-D}}}$.

So by (10), $\boxed{S = \sqrt{\frac{2C_2C_3}{C_4(C_3+C_4)}} \times \sqrt{\frac{D(I-D)}{I}}}$.

Further, $\frac{\partial^2 C}{\partial Q^2}\frac{\partial^2 C}{\partial S^2} - \frac{\partial^2 C}{\partial Q \partial S} > 0$ at $Q = \sqrt{2C_2\frac{C_3+C_4}{C_3C_4}} \times \sqrt{\frac{DI}{I-D}}$, $S = \sqrt{\frac{2C_2C_3}{C_4(C_3+C_4)}} \times \sqrt{\frac{D(I-D)}{I}}$.

The optimal order quantity (or, lot size) (say Q_{opt}) is $\boxed{Q_{opt} = \sqrt{2C_2\frac{C_3+C_4}{C_3C_4}} \times \sqrt{\frac{DI}{I-D}}}$.

The optimal cycle length (say t_{opt}) is $\boxed{t_{opt} = \frac{Q_{opt}}{D} = \sqrt{2C_2\frac{C_3+C_4}{C_3C_4}} \times \sqrt{\frac{I}{D(I-D)}}}$.

The optimal shortage (say $SH_{opt} = Dt_3 + (I - D)t_4$, now use the relation at point H) is $\boxed{SH_{opt} = 2Dt_3 = 2S_{opt} = 2\sqrt{\frac{2C_2C_3}{C_4(C_3+C_4)}} \times \sqrt{\frac{D(I-D)}{I}}}$.

The optimal cost per cycle (say C_{opt}) is $\boxed{C_{opt} = DC_1 + \frac{DC_2}{Q_{opt}} + \frac{[\frac{I-D}{I}Q_{opt} - S_{opt}]^2 C_3}{2Q_{opt}}\frac{I}{(I-D)} + \frac{S_{opt}^2 C_4}{2Q_{opt}}\frac{I}{(I-D)}}$.

4.3 Quantity Discount Model

◘ Here the quantity cost reduced if larger quantity ordered. This is also known as quantity discount or, price break models. For example, if we buy 5000 pens then the cost Rs. 10 per pen, for 5001 to 8000 the cost is Rs. 9.5 per pen and 8000 – 15000 the cost is Rs. 9.25 per pen.

► Now in each cycle what will be the optimal quantity unit such that maximum quantity discount avail with the minimum storage cost. Also the optimum cycle lenth is equally important.

4.3.1 Discount EOQ Model - I [A] (No Shortage)

◆ [A]: Demand Rate Uniform, Instant Replenishment.

An organisation must control its inventory for smooth performance. A basic inventory situation focouses on stock levels that are decreasing with time and stock refill with new units. A simple represenation of this situtaion can be explained through economic order quantity (EOQ) model. The EOQ models are based on some elementary assumptions.

◘ Assumptions: The main assumptions of classical EOQ models are

1. A known constant demand rate (D).
2. Instantaneous order (quantity Q) replenishment (restoration of a stock) i.e. replenish rate infinite. In other words, lead time is 0 i.e. instantaneous order placed and received.

3. No shortages are allowed i.e. Inventory cost = Purchase + Setup + Holding. □

The objective of EOQ model is to determine the frequency of order i.e. when and by how much (i.e. number of unit) we should order the inventory so that it minimizes the sum of these costs per unit time.

◘ Objective: The main target of classical EOQ models are to determine

1. The frequency of order i.e. optimal ordering cycle length (t).
2. The number of unit in each order i.e. optimum order quantity units (Q) at each ordering cycle. □

Suppose units of the product under consideration are assumed to be withdrawn from inventory continuously at a known constant rate D per unit time (Assumption 1). It is further assumed that inventory is replenished when needed by ordering a batch of fixed size (Q units), where all Q units arrive simultaneously (Assumption 2) at the desired time (Assumption 3). The time between consecutive replenishments of inventory is referred to as a cycle (t). For the basic EOQ model, the only costs to be considered are: C_1 = Purchase cost per unit (later we will modify as quantity discount), C_2 = Ordering cost (per order), C_3 = Holding cost per inventory per time. For the fixed demand rate, shortages can be avoided by replenishing inventory each time the inventory level drops to zero (See Figure 4.7), and this also will minimize the holding cost. We assume the cost of the items are not constant over time i.e. there are quantity discounts.

From the schematic diagram (Figure 4.7) it is observed that, x-axis represents time and y-axis represents inventory level. At the begining, we start fixed Q units (see the line OB) of inventory and as time goes, the inventory decreases with a constant demand rate D i.e. at an instance t_1 the $(Q - D \times t_1)$ units of inventory present in the store. Also the inventory units goes to zero at the end of the cycle (i.e. after time t) it implies $Q - D \times t = 0 \Rightarrow \boxed{Q = Dt}$. So the total holding inventory units during the time t i.e. the cycle length OA is = the area of the triangle OAB = $\frac{Qt}{2}$.

At the point A i.e. after the first cycle having length t, the inventory goes to zero and instant restoration the stock with Q units (see the line AB_1). The process continues upto n orders.

So, the various costs for the first cycle (i.e. per cycle) are:
Purchase cost = Item units × Cost per item = QC_1, Setup Cost = C_2, Holding Cost = Inventory units × Cost per item = $\frac{QtC_3}{2}$.
So the total cost per cycle = $QC_1 + C_2 + \frac{QtC_3}{2} = DC_1t + C_2 + \frac{DC_3t^2}{2}$.
Hence, the total cost per unit time is $C = DC_1 + \frac{C_2}{t} + \frac{DC_3t}{2} = DC_1 + \frac{C_2D}{Q} + \frac{C_3Q}{2}$.

Now we will minimize C with respect to t, $\frac{dC}{dt} = 0 \Rightarrow -\frac{C_2}{t^2} + \frac{DC_3}{2} = 0 \Rightarrow \boxed{t = \sqrt{\frac{2C_2}{DC_3}}}$.

Further, $\frac{d^2C}{dt^2} > 0$ at $t = \sqrt{\frac{2C_2}{DC_3}}$.

The optimal time interval of ordering or, cycle length (say t_{opt}) is $\boxed{t_{opt} = \sqrt{\frac{2C_2}{DC_3}}}$.

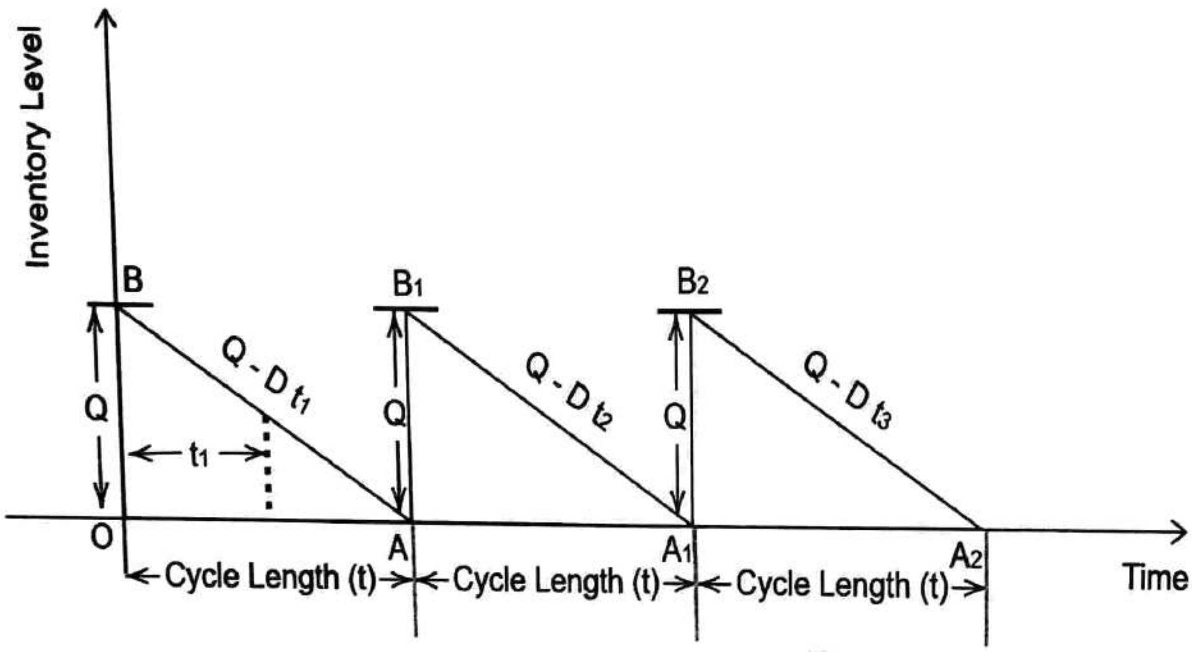

Figure 4.7: Inventory Situation with uniform demand rate and instantaneous order replenishment.

The optimal order quantity (or, lot size) (say Q_{opt}) is $\boxed{Q_{opt} = Dt_{opt} = \sqrt{\frac{2C_2D}{C_3}}}$.

□ **Inclusion of Quantity Discount :** Suppose there is a quantity discount i.e. the purchase cost C_1 vary with respect to Q. Suppose for purchasing $[0, Q_1)$ unit the price is C_{11} per unit, $[Q_1, Q_2)$ unit the price is C_{12} per unit, $[Q_2, \infty)$ unit the price is C_{13} per unit. Here $0 < Q_1 < Q_2$ and $C_{11} > C_{12} > C_{13}$.
So we can write,

$$C_1 = \begin{cases} C_{11} & \text{if } Q \in [0, Q_1) \\ C_{12} & \text{if } Q \in [Q_1, Q_2) \\ C_{13} & \text{if } Q \in [Q_2, \infty) \end{cases}$$

We know, the total cost per unit time is $C = DC_1 + \frac{C_2D}{Q} + \frac{C_3Q}{2}$.

Hence,

$$C = \begin{cases} DC_{11} + \frac{C_2D}{Q} + \frac{C_3Q}{2} & \text{if } Q \in [0, Q_1) \\ DC_{12} + \frac{C_2D}{Q} + \frac{C_3Q}{2} & \text{if } Q \in [Q_1, Q_2) \\ DC_{13} + \frac{C_2D}{Q} + \frac{C_3Q}{2} & \text{if } Q \in [Q_2, \infty) \end{cases}$$

Now the optimal order quantity is $Q_{opt} = \sqrt{\frac{2C_2D}{C_3}}$ for all the prices (as Q_{opt} are

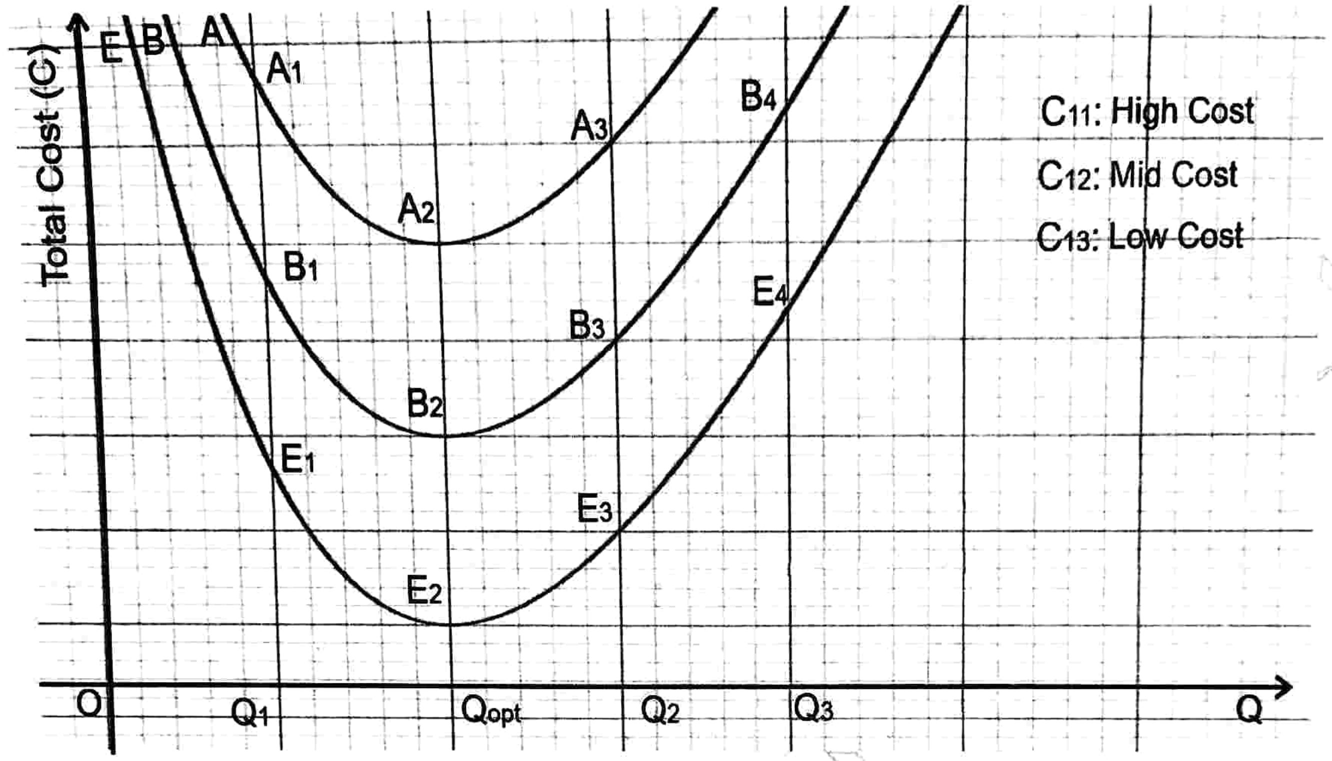

Figure 4.8: Inventory Situation with uniform demand rate and instantaneous order replenishment. Quantity discount applicable with three rates $C_{11} = 11$ (*Green*), $C_{12} = 10$ (*Blue*), $C_{13} = 9$ (*Red*). The other parameter values are $D = 1, C_2 = 20, C_3 = 5$,

independent of C_1). So there are three possibiliteis : i) $Q_{opt} \in (0, Q_1]$, ii) $Q_{opt} \in (Q_1, Q_2]$, iii) $Q_{opt} \in (Q_2, Q_3]$.
The situation is graphically explained by taking some hypothetical values of the parameters (see Fig. 4.8).

▶ Note that, cost function C minimizes at Q_{opt} i.e. either $AA_1A_2A_3$ (green) curve or, $BB_1B_2B_3B_4$ (blue) curve or, $EE_1E_2E_3E_4$ (red) curve satisfies the Q_{opt}.

▶ Case I: If $Q_{opt} \in [0, Q_1)$
So the Q_{opt} is valid for the quantity discount cost C_{11} i.e. total cost for C_{11} and Q_{opt} is $C_\alpha = DC_{11} + \frac{C_2 D}{Q_{opt}} + \frac{C_3 Q_{opt}}{2}$. However, we also need to check the total cost for the next two available discount price C_{12}, C_{13} with the minimum quantity Q_1 i.e. $C_\beta = DC_{12} + \frac{C_2 D}{Q_2} + \frac{C_3 Q_2}{2}$ and $C_\gamma = DC_{13} + \frac{C_2 D}{Q_2} + \frac{C_3 Q_2}{2}$.
Next we select Q_{opt} or, Q_2 or, Q_3 at which $\min\{C_\alpha, C_\beta, C_\gamma\}$ occurs. Hence either Q_{opt} or, Q_1 or, Q_2 will be the optimum order quantity.

▶ Case II: If $Q_{opt} \in [Q_1, Q_2)$
So the Q_{opt} is valid for the quantity discount cost C_{12} blue curve (Fig. 4.8) i.e. total cost for C_{12} and Q_{opt} is $C_\alpha = DC_{12} + \frac{C_2 D}{Q_{opt}} + \frac{C_3 Q_{opt}}{2}$. However, we also need to check the total cost for the next available discount price C_{13} with the minimum quantity Q_2 i.e. $C_\beta = DC_{13} + \frac{C_2 D}{Q_2} + \frac{C_3 Q_2}{2}$.
Next we select Q_{opt} or, Q_2 at which $\min\{C_\alpha, C_\beta\}$ occurs. Hence either Q_{opt} or, Q_2 will be

the optimum order quantity.

▶ Case III: If $Q_{opt} \in [Q_2, \infty)$

So the Q_{opt} is valid for the quantity discount cost C_{13} i.e. total cost for C_{13} and Q_{opt} is $C_\alpha = DC_{12} + \frac{C_2 D}{Q_{opt}} + \frac{C_3 Q_{opt}}{2}$. Hence Q_{opt} will be the optimum order quantity.

☐ In a similar way, it can be generalized upto k breaks for the discount cost C_1.

Example 4.3. *Find the optimal order quantity for a product when the annual demand for the product is 500 units. The storage (holding) cost per unit per year is 10% of the unit cost and the ordering cost per order is Rs. 180. The unit costs (in Rs.) are [JNTU 2011]*

$$C_1 = \begin{cases} 25 & if Q \in [0, 500) \\ 24.80 & if Q \in [500, 1500) \\ 24.60 & if Q \in [1500, 3000) \\ 24.40 & if Q \in [3000, \infty) \end{cases}$$

☐ *It is given that, Annual demand rate i.e.* $D = 500$, *Ordering cost per order i.e.* $C_2 = 180$ *and holding cost per unit per year i.e.* $C_3 = 0.1 \times$ *unit cost.*

Now for unit price 24.4, we have $Q_{opt} = \sqrt{\frac{2C_2 D}{C_3}} = \sqrt{\frac{2\times180\times500}{0.1\times24.4}} = 271.6$.

This is not feasible as $Q = 271.6$ *is not available for cost 24.4.*

For unit price 24.6, we have $Q_{opt} = \sqrt{\frac{2\times180\times500}{0.1\times24.6}} = 270.5$, *(not feasible).*

For unit price 24.8, we have $Q_{opt} = \sqrt{\frac{2\times180\times500}{0.1\times24.8}} = 269.4$, *(not feasible).*

For unit price 25, we have $Q_{opt} = \sqrt{\frac{2\times180\times500}{0.1\times25}} = 268.3$, *(feasible).*

Note that, Q_{opt} *is the value at which total cost* C *minimizes.*

So the total annual cost for 268.3 unit with price 25 is $C^1 = DC_{11} + \frac{C_2 D}{Q^1_{opt}} + \frac{C_3 Q^1_{opt}}{2} = 500 \times 25 + \frac{180\times500}{268.3} + \frac{0.1\times25\times268.3}{2} = 13,170.82$.

We also compare the above price with minimum quantity with next lower price break i.e. the total annual cost for 500 unit with price 24.8 is $C^2 = DC_{12} + \frac{C_2 D}{Q_2} + \frac{C_3 Q_2}{2} = 500 \times 24.8 + \frac{180\times500}{500} + \frac{0.1\times24.8\times500}{2} = 13,200$.

We again compare the above price with minimum quantity with next lower price break i.e. the total annual cost for 1500 unit with price 24.6 is $C^3 = DC_{13} + \frac{C_2 D}{Q_3} + \frac{C_3 Q_3}{2} = 500 \times 24.6 + \frac{180\times500}{1500} + \frac{0.1\times24.6\times1500}{2} = 14,205$.

Finally, we compare the above price to the minimum quantity with next lower price break i.e. the total annual cost for 3000 unit with price 24.4 is $C^4 = DC_{14} + \frac{C_2 D}{Q_4} + \frac{C_3 Q_4}{2} = 500 \times 24.4 + \frac{180\times500}{3000} + \frac{0.1\times24.4\times3000}{2} = 15,890$.

As the minimum cost is 13,170.82 *it implies the optimum order quantity is* 268.3 *i.e.* 269 *units.*

Example 4.4. *Find the optimal order quantity for a product for which the price breaks as follows [CheU 2002]*

$$C_1 = \begin{cases} 10 & if Q \in [0, 500) \\ 9.25 & if Q \in [500, 750) \\ 8.75 & if Q \in [750, \infty) \end{cases}$$

The monthly demand for the product is 200 *units, storage cost is* 2% *of the unit cost and cost of ordering is Rs.* 100.

☐ *It is given that, Annual demand rate i.e.* $D = 200$ *units, Ordering cost per order i.e.* $C_2 = 100$ *and holding cost per unit per month i.e.* $C_3 = 0.02 \times$ *unit cost.*
Now for unit price 8.75, *we have* $Q_{opt} = \sqrt{\frac{2C_2D}{C_3}} = \sqrt{\frac{2\times100\times200}{0.02\times8.75}} = 478.1.$
This is not feasible as $Q = 478.1$ *is not available for cost Rs.* 8.75.
For unit price 9.25, *we have* $Q_{opt} = \sqrt{\frac{2\times100\times200}{0.02\times9.25}} = 465$, *(not feasible).*
For unit price 10, *we have* $Q_{opt} = \sqrt{\frac{2\times100\times200}{0.02\times10}} = 447.2$, *(feasible).*
Note that, Q_{opt} *is the value at which total cost* C *minimizes.*
So the total annual cost for 447.2 *unit with price* 10 *is* $C^1 = DC_{11} + \frac{C_2D}{Q^1_{opt}} + \frac{C_3Q^1_{opt}}{2} = 200 \times 10 + \frac{100\times200}{447.2} + \frac{0.02\times10\times447.2}{2} = 2089.44.$
We also compare the above price with minimum quantity with next lower price break i.e. the total annual cost for 500 *unit with price* 9.25 *is* $C^2 = DC_{12} + \frac{C_2D}{Q_2} + \frac{C_3Q_2}{2} = 200 \times 9.25 + \frac{100\times200}{500} + \frac{0.02\times9.25\times500}{2} = 1936.25.$
Finally, we compare the above price to the minimum quantity with next lower price break i.e. the total annual cost for 3000 *unit with price* 24.4 *is* $C^4 = DC_{14} + \frac{C_2D}{Q_4} + \frac{C_3Q_4}{2} = 200 \times 8.75 + \frac{100\times200}{750} + \frac{0.02\times8.75\times750}{2} = 1842.29.$
As the minimum cost is 1842.29 *it implies the optimum order quantity is* 750 *units.*

4.4 ABC Inventory System

☐ ABC analysis (or, Always Better Control Analysis) is an inventory management technique that determines the value of inventory items based on their importance to the business. ABC ranks items on demand, cost and risk data, and inventory mangers group items into classes based on those criteria. This helps business leaders understand which products or services are most critical to the financial success of their organization.
▶ The most important stock keeping units (SKUs), based on either sales volume or profitability, are Class A items, the next-most important are Class B and the least important are Class C. Some companies may choose a classification system that breaks products into more than just those three groups (A-F, for example).

4.4.1 Pareto's Principle (The 80/20 Rule)

☐ The Pareto principle was developed by Italian economist Vilfredo Pareto in 1896. Pareto observed that 80% of the land in Italy was owned by only 20% of the population. He also witnessed this happening with plants in his garden 20% of his plants were bearing 80% of the fruit. This relationship is best mathematically described as a power law distribution between two quantities, in which a change in one quantity results in a relevant change into

the other.
▶ Some examples based on Pareto's Principle:
▷ 80% of a company's profits come from 20% of customers.
▷ 20% of a plant contains 80% of the fruit.
▷ 80% of exam preparation is done at last 20% of the study holidays.
▷ 80% of all wealth in the world lies with only 20% of the population.
▷ 20% of grocery items amounts to 80% total bill.
▷ 20% of the sports people win 80% of matches.
▷ 80% of the crimes are committed by 20% of the population.
▶ The 80/20 rule is not a formal mathematical equation, but more a generalized phenomenon that can be observed in economics, business, time management, and even sports.
◘ The basis of the Pareto principle states that 80% of results come from 20% of actions. If you have any kind of work that can be segmented into smaller portions, the Pareto principle can help you identify what part of that work is the most influential.
▶ Imagine you work at an ecommerce company. You take a look at 100 of your most recent customer service complaints, and notice that the bulk of the complaints come from the fact that customers are receiving damaged products. Your team calculates the amount of refunds given for your damaged products and finds that approximately 80% of refunds given were for damaged products. Your company wants to avoid processing refunds for broken products, so you make this problem a priority solution. Your team decides to update packaging to protect your products during shipping, which resolves the issue of customers receiving damaged products. Here we have used Pareto's principle for decision making.

Question 4.3. *How ABC Analysis Relates to the Pareto Principle?*
★ *The Pareto Principle says that most results come from only 20% of efforts or causes in any system. Based on Pareto's 80/20 rule, ABC analysis identifies the 20% of goods that deliver about 80% of the value.*
Therefore, most businesses have a small number of 'A' items, a slightly larger group of 'B' and a big group of 'C' items. The Pareto Principle may not always be completely accurate. However, analysis shows that valuable things do tend to bend toward an 80/20 distribution. ABC analysis identifies the 'A' items where most of a business's revenue comes from with relatively little effort. □

4.4.2 ABC Analysis

◘ Conduct ABC inventory analysis by multiplying the annual sales of a certain item by its cost. The results tell you which goods are high priority and which yield a low profit, so you know where to focus human and capital resources.

Question 4.4. *Write short notes on ABC Analysis.*
★ *The ABC analysis is based on Paretos law that a few high usage value items constitute a major part of the capital invested in inventories, whereas bulk of items having low usage value constitute insignificant part of the capital.*

It classify all the inventory items into three categories based on their usage values. Items of high usage value but small in number are classified as 'A' items and would be under strict control of top level management. Items of moderate value and size are classified as 'B' items and would attract reasonable attention of the middle level management and 'C' items are large in number but require little capital and would be under simple control. ABC analysis is also known as 'proportional value analysis'.
Usually, inventory items in most organisations show the following distribution patterns:
A : 5 − 10% of the total number of items accounting for 70 − 80% of the annual usage value,
B : 10 − 20% accounting for about 15 − 20% of the annual usage value, and
C : 70 − 80% of the number of items accounting for 5 − 15% of the annual usage value.
In view of the several hundred and even thousand items stocked in most inventory situations, the ABC analysis may be carried out on a sample. Once a random sample has been obtained, the following steps may be performed for the ABC analysis:

1. *Annual Usage Value: Calculate the annual usage value for every item in the sample: Annual Usage Value = Annual Requirement Units × Unit Cost.*
2. *Ordering: Arrange the above Annual Usage Values in descending order.*
3. *Cumulative Annual Usage & Items: Next, the cummulative total number of items and the annual usage value.*
4. *Cumulative Percentage Annual Usage & Items: Convert the cummulative totals of items and annual usage values into percentages.*
5. *Plot Lorentz Curve: Plot the two cumulative percentages. The curve obtained is called ABC distribution curve or, Pareto curve or, Lorentz curve.*
6. *Select Cut-off Points in Lorentz Curve: Mark the cut-off points X and Y where the curve changes its slope, dividing it into three segments A, B and C. These segments A, B and C for the sample are then generalised over the entire population of stock items.*

Under ABC analysis, an organisation would devote much time and effort in the control of 'A' items. It has higher inventory costs and be procured in smaller lots. 'B' items are usually placed under statistical control and attract periodic control of the management. (s, S) inventory control system might be used for these items. 'C' items require very little capital and hence have low inventory carrying costs and should be bought in bigger lots so that there are fewer orders and hence lower acquisition cost and also to take advantage of quantity discounts for bulk purchases. A fixed-order quantity system may be used for such items. □

Example 4.5. *Twelve items kept in inventory are listed below. Which items should be classified as A, B and C items? What percentage of items is in each class? What percent-*

age of total annual value is in each class? [CA 1983]

Items with required units and cost per unit					
Item No.	*Units*	*Cost (Rs.)*	*Item No.*	*Units*	*Cost (Rs.)*
1	7000	5	7	60000	0.2
2	24000	3	8	3000	3.5
3	1500	10	9	300	8
4	600	22	10	29000	0.4
5	38000	1.5	11	11500	7.1
6	40000	0.5	12	4100	6.2

⇒ *We will perform the following steps for the ABC analysis:*

1. Annual Usage Value : Calculate the annual usage value for every item in the sample:

Annual Usage Value = Annual Requirement Units × Unit Cost

2. Ordering : Arrange the above Annual Usage Values in descending order.

3. Cumulative Annual Usage & Items : Next, the cummulative total number of items and the annual usage value.

4. Cumulative Percentage Annual Usage & Items : Convert the cummulative totals of items and annual usage values into percentages.

5. Plot Lorentz Curve : Plot the two cumulative percentages. The curve obtained is called ABC distribution curve or, Pareto curve or, Lorentz curve.

6. Select Cut-off Points in Lorentz Curve : Mark the cut-off points X and Y where the curve changes its slope, dividing it into three segments A, B and C. These segments A, B and C for the sample are then generalised over the entire population of stock items.

First we will perform steps 1, 2 in the following table:

Item No.	*Units*	*Cost (Rs.)*	*Annual Usage*	*Ranking*
1	7000	5	35000	4
2	24000	3	72000	2
3	1500	10	15000	7
4	600	22	13200	8
5	38000	1.5	57000	3
6	40000	0.5	20000	6
7	60000	0.2	12000	9
8	3000	3.5	10500	11
9	300	8	2400	12
10	29000	0.4	11600	10
11	11500	7.1	81650	1
12	4100	6.2	25420	4

Now to perform steps 3, 4 we first write the table with respect to ranking and then find cumulative and cumulative percentage.

Item No.	*Annual Usage*	*Cumulative Annual Usage* (C_3)	*Percentage Annual Usage* ($C_4 = C_3/355770$)	*Cumulative Items*	*Percentage Items* (C_6)
11	81650	81650	22.95	1	8.33
2	72000	153650	43.18	2	16.66
5	57000	210650	59.20	3	25
1	35000	245650	69.04	4	33.33
12	25420	271070	76.19	5	41.66
6	20000	291070	81.81	6	50
3	15000	306070	86.03	7	58.33
4	13200	319270	89.74	8	66.66
7	12000	331270	93.11	9	75
10	11600	342870	96.37	10	83.33
8	10500	353370	99.32	11	91.66
9	2400	355770	100	12	100

Next to perform step 5 i.e. to draw Lorentz curve, we plot percentage items (C_6) in x-axis and percentage annual usage (C_4) in y-axis. Then we connected the points to visualize the curve (see Fig. 4.9). To perform step 6, we have identified the cut-off points X and Y where the curve changes its slope, dividing it into three segments A, B and C. Here the item number 11, 2, 5 are classified as A, item number 1, 12, 6, 3 are classified as B and item number 4, 7, 10, 8, 9 are classified as C. In the above table we have classified these three groups as red, green and blue colours.

□ *Using the above table we observe that: In class A, 25% items present; In class B, 33.33% (= 58.33 − 25) items present; In class C, 41.67% (= 100 − 58.33) items present;*

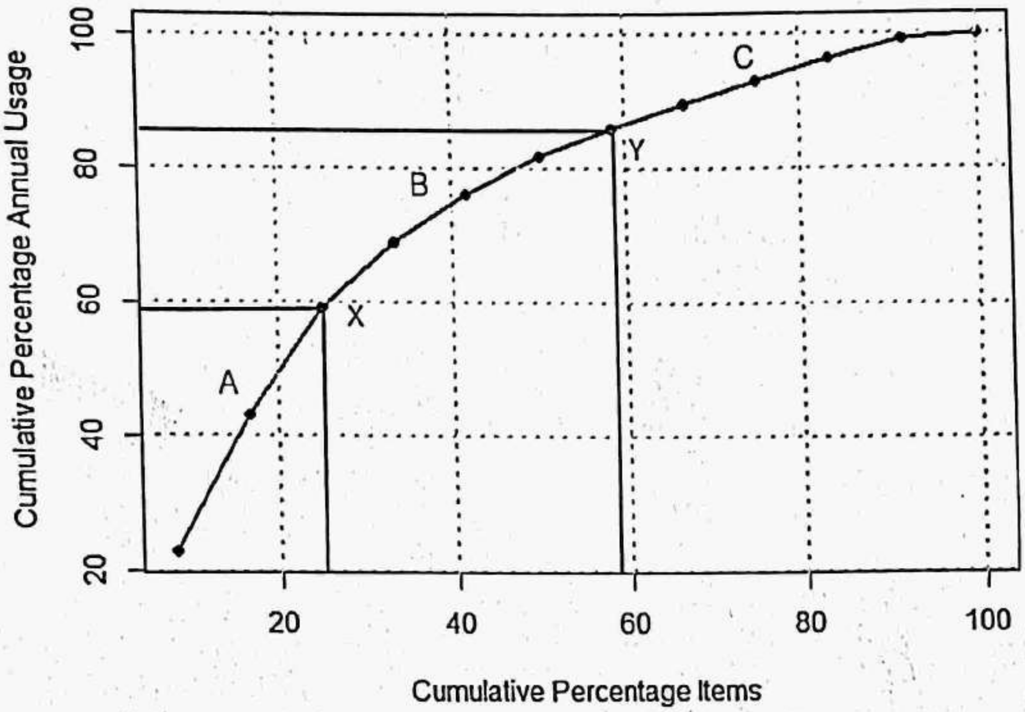

Figure 4.9: Lorentz curve for the example 4.5.

□ *Using the above table we observe that: In class A,* 59.2% *of total annual value is present; In class B,* 26.83% (= 86.03 − 59.20) *items present; In class C,* 13.97% (= 100 − 86.03) *items present;*

[Do It Yourself] 4.2. *The following information is known about a group of items. Classify the items as A, B and C. [CA 1980]*

Items with annual consumption units and cost per unit					
Item No.	*Units*	*Cost (Rs.)*	*Item No.*	*Units*	*Cost (Rs.)*
1	*30000*	*0.1*	*6*	*220000*	*0.1*
2	*280000*	*0.15*	*7*	*15000*	*0.05*
3	*3000*	*0.1*	*8*	*80000*	*0.05*
4	*110000*	*0.05*	*9*	*60000*	*0.15*
5	*4000*	*0.05*	*10*	*8000*	*0.1*

[Do It Yourself] 4.3. *The following information is known about a group of items. Which items should be classified as A, B and C items? What percentage of items is in each class? What percentage of total annual value is in each class?. [ICWA 1991]*